DISCRETE FOURIER TRANSFORM

- DTFT is

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}.$$

DTFT is not suitable for DSP applications because

- In DSP, we are able to compute the spectrum only at specific discrete values of omega.
- Any signal in any DSP application can be measured only in a finite number of points.

- DFT is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}} \quad \text{analysis}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{kn}{N}} \quad \text{synthesis.}$$

- Alternative formulation of DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) W^{kn} \quad \longleftarrow W = e^{-j\frac{2\pi}{N}}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W^{-kn}.$$

- Y = fft(X,n)

```
n = [0:29];
x = cos(2*pi*n/10);
N1 = 64;
N2 = 128;
N3 = 256;
X1 = abs(fft(x,N1));
X2 = abs(fft(x,N2));
X3 = abs(fft(x,N3));
F1 = [0 : N1 - 1]/N1; % 0 to N-1→frequency, normalized
F2 = [0 : N2 - 1]/N2;
F3 = [0 : N3 - 1]/N3;
subplot(3,1,1)
plot(F1,X1,'-x'),title('N = 64'),axis([0 1 0 20])
subplot(3,1,2)
plot(F2,X2,'-x'),title('N = 128'),axis([0 1 0 20])
subplot(3,1,3)
plot(F3,X3,'-x'),title('N = 256'),axis([0 1 0 20])
```

```
n = [0:149];
x1 = cos(2*pi*n/10);
N = 2048;
X = abs(fft(x1,N));
X = fftshift(X); %Shift zero-frequency
  component to center of spectrum
F = [-N/2:N/2-1]/N;
figure(1)
plot(F,X)
xlabel('frequency / f s')
figure(2)
plot(X)
```

- ezplot('1/(x^2+2)') long tail
- ezplot('exp(-x^2)') bell shaped